III. Analyzing type II error (β)

A. Type II error is the probability of accepting a false null hypothesis.

B. Linda's two-tailed study concerning any change in the average purchase price from last year's \$7.75 (see page 85) will be analyzed. First we will calculate the lower critical value, an accept/reject point for this null hypothesis.

$$\begin{array}{c} \mu - z(\sigma_{\overline{x}}) \\ \alpha + 2 = .01 + 2 = .005 \\ .50 - .005 = .495 \rightarrow z = \pm 2.58 \end{array}$$
 \$7.75 - 2.58 (\$.10) \$7.75 - \$.258 = \$7.49

1. Here, type II error exists everywhere except for $\mu = 7.75 .

2. This means the amount of type II error varies depending upon the value of the true population mean.

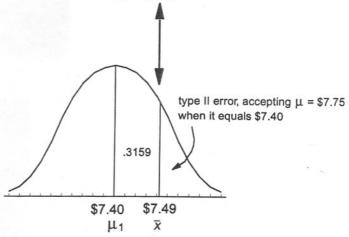
 We will calculate the probability of type II error for a population mean (μ₁) of \$7.40.

$$Z = \frac{\overline{x} - \mu_1}{\frac{\sigma}{\sqrt{n}}}$$

$$Z = \frac{\$7.49 - \$7.40}{\frac{\$.70}{\sqrt{49}}} = \frac{\$.09}{\$.10} = .90 \rightarrow .3159$$

type II error is .50 - .3159 = .1841

When the mean is \$7.40, Linda's decision rule has a type II error of 18.41%.



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Accept

\$7.75

Reject

.005

z = +2.58

Reject

.005

 $\bar{x} = \$7.49$

z = -2.58

C. Operating characteristic curves

 The operating characteristic curve graphs the probability of type II error. It depicts all possible type II errors given some acceptable level of type I error. It measures accepting no change when there has been change.

 As the true population mean in the above example drops, accepting a false null hypothesis becomes less likely as the right tail area of the second graph becomes smaller. Eventually the true population mean is so small that accepting a false null hypothesis is almost impossible.

3. As the true mean approaches \$7.75, the area to the right gets larger. It reaches a peak of 98+ percent just before \$7.75. Type II error does not exist for $\mu = \$7.75$ because the null hypothesis is not false.

 At a point just beyond \$7.75, beta error is still 98+ percent and it drops toward zero as the true population mean increases.



 A power curve graphs the probability of not making a type II error. It measures:

a. how often you correctly reject a false null hypothesis

b. how often you accept a correct research hypothesis

2. It is the complement of type II error or 1 - type II error.

3. The power curve shows accepting a change in quality, consumer attitude, and voter preference when there has been changes in these areas.

 Lowering type II error comes at the expense of increasing type I error and vice versa.

